

Energy Efficient Modeling of Wireless Sensor Networks using Random Graph Theory

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Abstract— This paper deals with the discussion of an innovative and a design for the efficient power management and power failure diagnosis in the area of wireless sensors networks. A Wireless Network consists of a web of networks where hundreds of pairs are connected to each other wirelessly. A critical issue in the wireless sensor networks in the present scenario is the limited availability of energy within network nodes. Therefore, making good use of energy is necessary in modeling a sensor network. In this paper we have tried to propose a new model of wireless sensors networks on a three-dimensional plane using the percolation model, a kind of random graph in which edges are formed between the neighbouring nodes. An algorithm has been described in which the power failure diagnosis is made and solved. The concepts of Electromagnetics, Wave Duality, Energy model of an atom is linked with wireless networks. A model is prepared in which the positioning of nodes of sensors are decided. Also the model is made more efficient regarding the energy consumption, power delivery etc. using the concepts of graph theory concepts, probability distribution.

Index Terms— wireless sensor networks, Probability Distribution function, Energy Consumption, GAF, percolation.

I. INTRODUCTION

Sensor networks are composed of a large number of tiny sensors nodes that include sensing, data processing and communicating components. These sensors autonomously establish connections via various efficient wireless communication techniques. Sensor networks have the tendency to gather the information in a certain area depends on its range by sensors and transmit it back to the observer through a sink node as defined in [4]. In any wireless network let Zigbee network, the quantities of the sensors present are very large as the area to monitor is also very large. It is quite difficult to arrange the sensors in proper manner by man power; it couldn't be able to monitor the entire region under its vicinity. So the sensors are arranged in the stochastic ways. These lead to the location of sensors and information on their neighbors being, unknowable before the network is established, which brings uncertainty and randomness into the network. Energy of the sensor is limited in sensor networks. Therefore, one of the major goals in investigating sensor networks is to propose new innovative ways through which the power should be efficiently managed as described in [2-3], energy consumption, new routing algorithms [5], topology control methods, special MAC protocols [6] and other techniques such as data aggregation [7-9].

The main aim in the paper is to create a model through which the energy efficiency of the system is increased. In order to create this model concept of probability, P.D.F., C.D.F. etc. are involved. The importance of percolation theory for phase transition is also important for increasing the energy efficiency in any wireless network see reference [14, 15] and the references there in. Ref. [16] has described regarding the

importance of phase transition phenomena through which better design of WSN's could be obtained. Statistical study of probability is taken through [18]. Ref [17] describes regarding the probability based direction and Omni-directional broadcast of the signals for bonding and also regarding the use of directional antennas which could be used for the easy transmission in the mobile networking. In this case a lot of duplicate packets of the signals are saved.

The paper consists of model which is based on the energy distribution concepts of atomic bonding. Here the concept of Electron model of the atom is involved. The concepts of Bohr's atomic model are used to make the network management in the system. The energy distribution of Maxwell equations are also used to make a model which is energy efficient.

Earth is elliptical in shape and the data need to be transferred from one place to other. For proper model we should divide the earth's lattice space to spherical lattice system having three dimensions. The varying parameters are radius considering the center line as the reference line, radial angle and apex angle. In the previous models described below square or any polygon lattice is used. In our model spherical lattice is considered and concepts of wireless networks are linked with other topics for efficient modeling.

In Ref. [19] it has been described regarding an algorithm in which several nodes are selected to be like a cluster which is based on the geographical position of the sensors. With this paper we brought the cases of probability density function in the areas having high sensing network and distributed the power as accordingly depending on the value of their P.D.F. or depending on the rate at which their C.D.F. increases over a particular geographical locations. The probability conditions involved are taken from the model described in the Ref. [1]. The basic ways through which the probability conditions of the random variable or a random graph is drawn and studied thoroughly. Also various techniques are involved which takes the cases of probability cases.

The case considered in this paper is a particular area which is having sensor nodes in the form of a function $G(S, E)$ where S denotes the sensors in that particular area being under vicinity and E denotes all the existing communication connections. A technique is involved in which the amount of nodes over an area where the signal is to be transmitted will be given with the power and the remaining nodes will be in the off stages. This will save a lot of power as a path will be selected (shown in fig. 2) and the signal is made to flow through that path only. Due to this there will be less case of cross polarization between the signal traveling.

The remainder of the paper is described in the following ways: Section II gives the brief introduction regarding the related work. In the section III, the model which must be prepared based on the probability density is provided. An improvement is provided regarding the power management is also given in the same section. Also some further research and problems are explained in section IV. Finally conclusion is in section V.

II. RELATED WORK

A. Geographical Adaptive Fidelity Algorithm:

GAF algorithm includes the selection of certain nodes based upon the geographical location of the nodes. The area which is to be monitored is divided into several small lattices forming a cluster. The size of the cluster also matters as it should meet the needs of the communication in which the signal must flow between two farthest possible nodes.

Assume a virtual lattice square with r units on a side and denote the radio range of the sensor by R . Therefore, we get

$$r^2 + (2r)^2 \leq R^2$$

Here the nodes are active and inactive periodically. The node has the tendency to exchange the information from its side node after becoming active and new cluster selection occurs. Cluster head is kept active all the times but the cluster members sleep in case there is no sensing task.

B. Site Percolation

It is a kind of random graph in which the edges are formed between two neighbouring nodes. In site percolation either node is considered as an open port with a probability p or a closed port with a probability of $(1-p)$ as described in [15-17]. Statistical analysis of probability is done in [18]. An edge exists only when there is a connection between two open ports of nodes. Site Percolations can be understood as a sudden change from the finite number of clusters to infinite number of clusters where p increases to p_c .

C. RANDOM GRAPH

A random graph consists of vertices and edges. Any two vertices share an edge with the probability of p . In Ref. [19] it has described that the probability of a random graph being connected tends to be one if the number of the edges taking part in the connection is higher than $P_c = (N/2 * \log N)$. Due to this there is large change in the system performance this is phase transition in the random graph theory. The value of P_c is like a threshold value for the probability distribution as if a certain network has this sought of probability distribution then that random graph is connected. This kind of mapping technique is quite unrealistic because we have wireless edges as connection. So fair pairs of vertices need to be created, sensor has limited communication radius.

D. BASIC DEFINITIONS

In the proposed model, a small model is taken into consideration and then this case is made to apply for large scale network. We take the case of a random graph containing origin, then considering the expected value of this network system when it will be large consisting infinite number of links. Let D_0 be the connected component containing the origin and denote the links by modulus of D_0 . Now taking this network to a large network we take the mathematical Expectation value E (modulus of D_0).

$$E(|D_0|) = \sum_{n=0}^{\infty} (nP |D_0| = n)$$

The expected value is in the form of increasing function of probability p , and will be available as equal to infinite when the value of probability goes to $p = 1$. Actually the value of P is a kind of threshold which is used to decide whether there could be a condition of well-connected network. One more condition which could be used for deciding the connection of network is the taking the set of infinite network and studying the expectations of the different cases.

III. MODELS (NAMED AS WIRELESS NODE BONDING MODEL)

A. Setting

Now we explain our model, our model deals with the atomic bonding model of the Bohr's Atomic Model. As in the atomic model there are electrons which have the tendency to revolve around the nucleus. The same concept is used in this model with some modification in order to have synchronization within the sensor networks. Here the nodes are kept stationary and they are assumed as electrons but they are not movable the thing that moves are electromagnetic waves. In the center we have a node which is the made primary transmitter, also around the primary node we have several revolving patterns over which the other nodes are present. The number of nodes in a particular orbit is decided by the formulae as defined in the Bohr's Atomic Distribution. The nodes in the orbits are decided also with the motto in order to have a high connective network which can occur if there is a proper synchronization within the network.

The radius of the circular orbits decided for the network model depends on the sensitivity of the networks. Therefore, generally the radius is calculated by considering the Probability distribution Function (P.D.F.) of the sensing network. The P.D.F. of the network will provide us the rate at which the probability is changing with increase in the distance from source node. The case is studied and it has been found to have the radius of the circular patterns as produced by the nodes of an antenna of any node is almost 0.75 times the difference between the radiuses of the circular orbits. The proof is given below from the following diagram:

Accordingly the radius goes on decreasing we go inside the orbits but the difference between the radiuses of the two orbits remains the same.

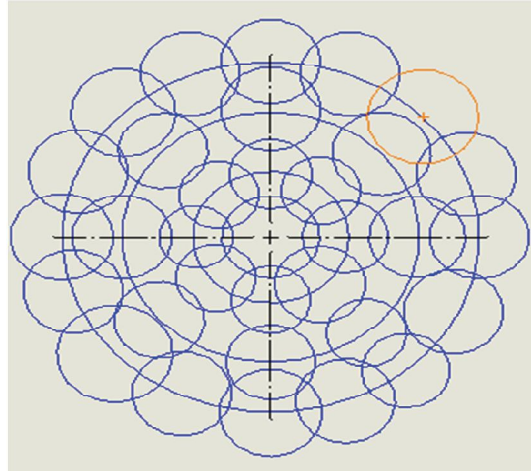
If the maximum radius of the circular patterns at each node is 0.75 times the difference in the radius of the circular orbits then the efficiency through which the data need to be transferred will be efficient. The other more important thing is to decide the distance between the orbits or the difference between the radiuses of the consecutive orbits. For this problem the solution is directly taken from the Bohr's Atomic Theory. The difference between the radiuses of the consecutive orbits of networks will be same as that for the electrons orbit as that for an atom.

Above figure explains about the proposed model of the wireless sensor network in which there are various number of nodes in the specified orbits. Also in this model, node is referred to the presence of electron.

In the case of Bohr's Atomic Model we have the energy dissipation as:

$$E = E' * \left(\frac{Z}{N}\right)^2$$

Here Z =Atomic No., N = Number of orbit,
 E' = Energy deviated by one hydrogen atom to bind the electron in its orbit.

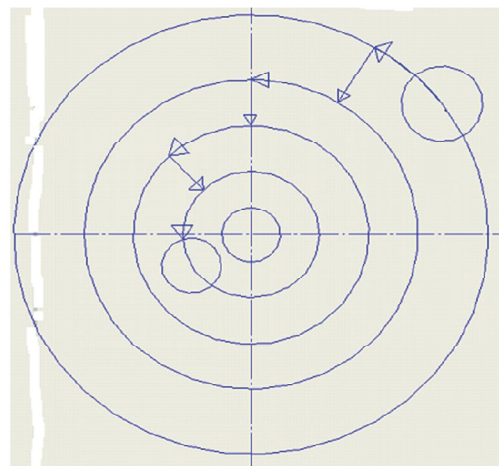


(Basic Display of model) Fig.1

Therefore, if we compare this formula with the case of wireless sensing then the energy deviation in the larger orbits will be higher than the lower orbits. By this we can say that the energy given to the nodes at higher orbit radius must be higher than of inner orbits.

The antenna used for transmission of the signals in the case of network model is taken to circular pattern and it has been clear that the initially when the signal wave has less radius (case of inner orbits) the concentration of the energy density is quite high. So the energy dissipated by the signal in the higher radius is high.

The concept explained above is valid for the wireless signal networks as well. Consider two nodes one in the third orbit and other one in the first orbit according to the orbit shown above. The signal from one node need to be transferred from one node to other, for this purpose line of sight transfer of signal can be used but it is power consuming and the nodes in that particular network should have high resolution power. In order to solve this problem the above model can be used which involves the transferring of the signal from one node to other as being transferred in the atom. In the atom, electrons having higher energy travel in the higher orbits but as the energy decrease the electrons shifts to the lower orbit. In this way, indirectly a way should be provided so that the signal should automatically go to lower orbits when required. Also the algorithm is developed to send the data from one node to another data in the form of energy efficient manner. As shown in the figure the signal should flow in the manner for efficient energy consumption.



(Flow of Signal in the model) Fig.2.

From the above diagram, efficient way for the transmission of signal is shown. Mathematically, energy required for moving of electron in a particular orbit is:

This relation is applied to our model for the calculation of energy in one orbit that is the energy required for the nodes that consists our sensors. The sensors on the outer orbit have the tendency to have the tendency to use lot of energy. In our model minimum energy which is required by a sensor network is shown. According to the Somerfield Theory, we have the energy required for an energy level is:

$$E = E' * \left(\frac{Z}{N}\right)^2$$

Therefore, taking the value of E' as 13.9 ev and N as no. of orbits. We calculate the power dissipated by a particular node to move through the arc of a particular nth orbit,

Power = (energy/time).

Therefore, to calculate the time for which signal will flow through nth orbit.

T= D/V

V= speed of light.

D = distance travelled by the signal over the arc

$$= (2 * D\pi * R * \left(\frac{\theta}{2\pi}\right))$$

Here θ is the angle made by the radial vector with the node. On simplifying we get

$$D = (\theta * R)$$

$$c = 3 * 10^8 \text{ m/sec}$$

$$T = (\theta * R) / 3 * 10^8.$$

Power dissipated for transferring the signal in nth orbit will be

$$P = E/T$$

$$P = [3 * 10^8 E' * Z^2 * \left(\frac{1}{N^2 * \theta * R}\right)]$$

Here

E' = energy dissipated when signal need to be travelled from the node on the first orbit to the center.

N = Orbit number.

θ = angle made by the radial vector of the node of the destination end.

R= Radius of the current orbit.

The given power is the power deviated by the signal when it will travel θ angle over an orbit of radius R. The value of Z will be termed as constants and the actual power will be inversely proportional to node number, angle and radius of the orbit.

Taking the case that if signal need to transferred from node n to node m then the power will be the sum of the total power as consumed by the sensor network for the transmission of data. Let Pi be the power required for transmission of data then total power Po:

$$Po = \sum_{i=m}^n Pi$$

The model includes the concept through which the signal should travel efficiently. Some rules of gravitation are also included in this paper. As in the case of a space craft in order to reach to certain planet(destination) it has the tendency to revolve other planet for gaining a proper speed and acceleration provided by the gravitational energy of the planets. By gaining this energy the space craft is well sufficient with its energy and can be used to reach its proper destination. In the same way the signals from the nodes should travel in the energy efficient way so the limited energy sensor network could be used with maximum efficiency. The sensor network at each node consists of antenna through which it could make the signal to transfer from one node to other. The way through which this signal energy should be transferred is through the same way as in the case of a space craft or electron. The signal of one node should be linked with adjacent node so this signal which could be weak in the mid-way can again be energized properly.

B. Considering probability point of view:

Consider a small network model with three orbits and let number of nodes in the first orbit is four. The probability distribution will involve the probability distribution density of the model and then it will be considered for a large network by the calculation of expectation of very high connected network.

Let the data needs to be transferred from one node to other and the magnetic lines coming out of Spherical lattice). The probability density will go on decreasing as the radius of the wave increases.

$P(c) \propto (1/(\text{radius of the magnetic waves}))$

As specified above the difference of the radius of orbits are dependable on the probability concepts, the value of the radius is derived from the probability. It has been found out that the strength of the signal is well enough to transfer the signal upto the probability density reaches upto P_c (next section); this means the equivalent radius difference of the orbits is double of the distance up to which the probability is P_c (as shown in the figure). This value of P_c has been proved to be ideally as 0.5 in ref.[1], they have included a square lattice model and studied the status of the nodes in the different probabilities of the network (signifying the strength of network).

C. Probability having infinite component:

Now consider the case of expectation of infinite number of orbits and magnetic waves generated by each node. Let the number be $|W_0|$, this means that range where the communication from the base stations is reachable and the number of stations within the range. In the practical case the total numbers of orbits and waves per node are finite but in the simple model, we have infinite number of orbits for covering a large network. When the component W_0 is infinite then the system can be considered as the system with high network connectivity. We set the probability of this component W_0 which is infinite to be $\theta(p)$, since it is a function of probability p . This distribution has been studied and found out with the graph shown below (fig.3 and fig.4). Initially the strength of the signal is quite high as the loss of energy is low in that area. However in the graph it has been shown that as the probability increases the probability function $\theta(p)$ rises after crossing a threshold probability p_H (mostly 0.5) Before this threshold value of $\theta(p)$ is zero but after crossing this value the slope of graph increases describing that there is a high connectivity in the network.

D. Overlapping acting as an advantage:

In the above proposed model, spherical lattices are taken and the wireless orbits are defined consisting of nodes depending of the type of network, these nodes emit circular waves to transmit data. There is always a case when there is an overlapping between two circular wave patterns. Generally this is taken as a problem as it could create the interference in the signal, but in our model it is acting as helping hand for increasing the probability distribution for those regions where the strength of the signals is low. The weak signal from one node and weak signal of other node could energize each other. As we know that the radio waves shows the property of mutual co-existence that they don't interfere during transmission.

If the area arrives as described when the probability of the signals is lower than p_H then then overlapping could be used for increasing the probability of the strength.

IV. IMPROVEMENT IN THE MODEL

By this model it can be identified about the most efficient way to transfer the data (explained earlier), meaning the algorithm for efficient node selection.

One another method that can be used for increasing the efficiency of the model is making only one node to be on with power supply and others to be in sleeping stage. Some sought of artificial intelligence should be included in the networks for the selection of the nodes for transferring data. This can also be done by matching the energy requirements of the network (explained in previous sections). Also in our model there is some energy compensation because the probability of high connecting network is intentionally kept low for increasing the life of sensor. From phase transition methods, it was found out that the value of p is 0.5 but if it is greater than one then there will be highly connected network. So, the efficiency of only one sensor should be high at one time and others should be shut with their power. Some other methods can also be easily included in the model such as power failure diagnosis.

The model should work in all parts, in all climates, in all situations so the selection of proper antenna is also essential. The radio wave attenuation is difficult to analyze inside the buildings therefore half wavelength dipole antenna should be used. In the case of half wave dipole antenna

$$E = -20 \log(d)$$

Where E denotes relative electromagnetic field strength and d is the distance between the transmitter and receiver. This shows the relation between then dissipated energy and the distance up to which it can have a good network connection.

V. SIMULATIONS

A. SCILAB Simulations:

In this case SCILAB simulations are included describing the change in the probability to the distance from the node. Probability Distribution Function (P. D. F.) and Complex Distribution Function (C. D. F.) of the system has been included.

Consider the case of a wireless sensor network having a primary node and circular orbits. Taking the case of one node and study has been made regarding the probability functions which are decreasing with the increase in radius of circular orbits. We have considered as the decrease in the probability is following a decreasing exponential pattern. Therefore, we take the case of Exponential Distribution.

$$(P.D.F.) \theta(p) = (1/\text{lam}) * (\exp(-\text{lam} * r))$$

Here the variables are r and θ where

r = radius of the circular orbit for each node.

$\theta(p)$ = exponential distribution function.

lam = constant value.

So by the exponential distribution we have the decreasing curve which represents the decreasing behavior of probability function with respect to the radius of the orbits, also it has been seen in the case of C.D.F. function initially there is a heavy increase in the function but after it crosses a certain radius value it becomes constant as one.

$$(C.D.F.) \theta(p) = (1 - \exp(-\text{lam} * x))$$

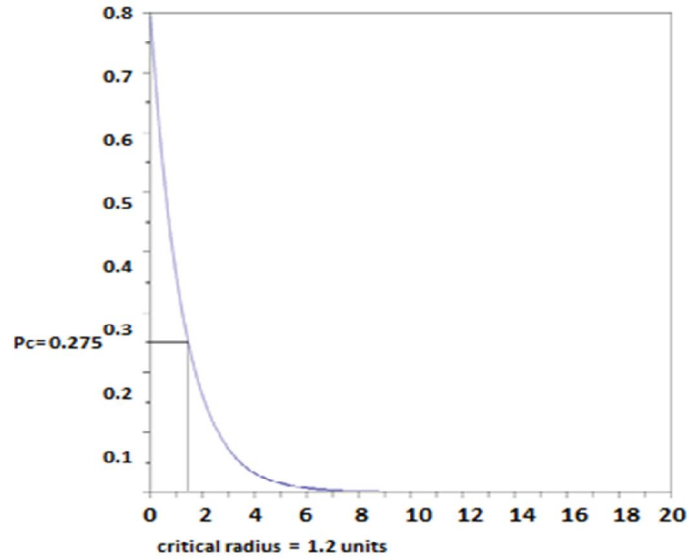
By the use of P.D.F. and C.D.F. graph it is observed that the value of CDF becomes equal to one fastly this shows that the probability density is higher in the region with lower radius and it decreases with the increase in the radius.

From the PDF graph it is clear that there is a decrease in the value of the probability density with the increase in the radius of the atom.

For $\text{lam} = 0.8$ we have taken the following simulations.

Yaxis denotes the C.D.F. probability function and x axis denotes the radius of the orbits.

Probability Distribution function

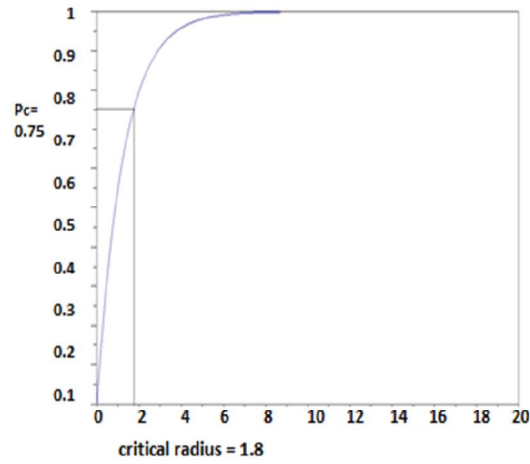


(Graph between probability density function and critical radius) Fig. 3

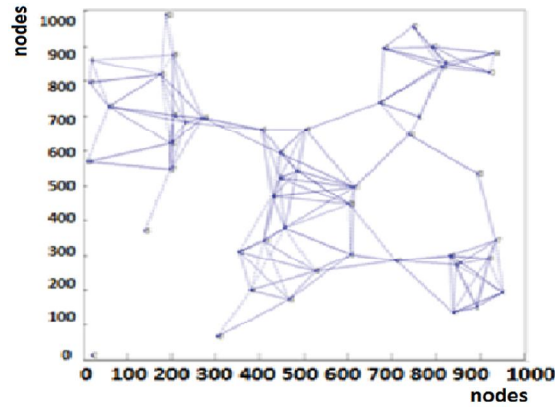
Yaxis denotes the C.D.F. probability function and x axis denotes the radius of the orbits.

Cumulative Probability Distributive Function

A MATLAB simulation is also given regarding the connection of nodes for transferring the signal from one node to another. In this case total 50 nodes are selected and made to connect if the probability between them for connection is more than 0.5. The nodes are selected randomly whether they are connected or not, simulation is given in fig. 5.



(Graph between the cumulative probability density function and critical radius)Fig. 4



(Graph representing selection of nodes for an 1000*1000 area)Fig. 5

B. Future Research:

As the model includes the concept of Bohr's Atomic Model and other concepts of Probability, so a lot of research work can be possible in this area consisting of the development of the energy distribution pattern. There is always a problem when you are increasing the energy efficiency of the system that is the time required for the system will be high. So some sort of more research should be done to eradicate this problem.

VI. CONCLUSION

We have referred the concept of random graph theory for making a model on wireless sensor network. The model uses percolation theory, a kind of random graph where edges are formed only between the nearby nodes. The model uses the concept of Bohr's Atomic Model. Several conditions regarding the use of probability distribution is involved for finding the areas having highly connected networks. The paper has also included techniques for which data can be efficiently transferred from one node to another. A lot of improvement is done over the model in order to make it energy efficient. A lot of research work can be done over this model through which it can be more made energy efficient.

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